AIAA Journal
Vol. 44, No. 2, February 2006

Technical Notes

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Fatigue Crack Growth Prediction Under Spectrum Load Using Crack Driving Force ΔK^*

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Nomenclature

a	=	crack length
C	=	Paris constant in fatigue crack growth equation
C_i	=	constant in fatigue crack growth equation, $i = 1, \ldots, 3$
da/dN	=	fatigue crack growth rate
K	=	stress intensity factor
K_{max}	=	maximum stress intensity factor
K_{min}	=	minimum stress intensity factor
K^*_{th}	=	threshold K^*
L	=	specimen length
m	=	Paris and Erdogan[4] exponent
N	=	total number of σ_{max} or σ_{min} in load sequence
N_r	=	total number of reversals in spectrum load block
R	=	stress ratio or load ratio
t	=	specimen thickness
W	=	specimen width
ΔK	=	stress intensity factor range
ΔK^*	=	Kujawski's[2] crack driving force parameter
ΔK_{eff}	=	effective stress intensity factor range
ΔK_{th}	=	threshold stress intensity factor range
ΔK^+	=	positive part of stress intensity factor range
σ_{max}	=	maximum stress
σ_{min}	=	minimum stress
σ^{rms}_{max}	=	root-mean-square maximum stress
σ^{rms}_{min}	=	root-mean-square minimum stress
σ_{UTS}	=	ultimate tensile strength
σ_y	=	yield strength

Introduction

THE damage tolerance concept is widely used in the aircraft industry to analyze the fatigue life of aircraft components. The total fatigue life of a component consists of life until a crack is initiated and life until this crack propagates to a critical size. Because of the existence of minor cracks at stress raisers in aging airframes, analysis of crack growth life assumes significant importance. Studies on fatigue crack growth behavior coupled with nondestructive test methods would help in determining inspection interval schedules for safe operation of the aircraft. In crack growth analysis, the spectrum load blocks required for a fatigue crack to grow from a given initial size to a critical size is evaluated based on well-established fracture mechanics principles.

A cycle-by-cycle approach for fatigue crack growth prediction under a spectrum load sequence is generally performed as follows: For the material under consideration, constant amplitude (CA) fatigue crack growth rate (FCGR) data are generated at various stress ratios R. This wide band of FCGR data is then merged to result in an almost single characteristic FCGR curve for the material by using either Elber's crack closure concept[1] or two mechanical crack driving force parameters[2,3] ΔK and K_{max}. Such a characteristic FCGR curve is approximated in an equation form by a Paris and Erdogan[4] type of law, $da/dN = C[f(\Delta K_{eff})$ or $f(\Delta K, K_{max})]^m$. In contrast, cycle counting, generally by a rain flow method, is used to separate spectrum sequences into individual load/stress cycles. The magnitude of crack extension for each of these rain flow counted cycles is calculated by the characteristic FCGR equation. The load sequence effects resulting in crack growth acceleration and/or retardation are accounted for while estimating the crack growth for each of these individual cycles by incorporating various load-interaction models.[5,6] The crack extension is calculated for every rain flow counted cycle in the spectrum load block, and the whole procedure is repeated to obtain crack length vs spectrum load block curves.

Though excellent analytical models using the crack closure concept have been developed to predict crack growth under spectrum loading, many uncertainties still exist in experimentally determining and analyzing crack opening/closure level, one of the important input parameters in analytical models for crack growth predictions. Hertzberg et al.[7] found that the crack closure was only partially effective in explaining the observed crack growth behavior. Careful experimental measurements have indicated that crack opening load P_{op} is not a unique value but depends on the measurement location and the technique employed.[8] Recently, it has been shown[9] that a significant contribution toward fatigue damage occurs in the load range even below the crack opening load. Meggiolaro and Pinho de Castro[10] found that crack closure cannot be used to explain the overload-induced retardation effects and suggested reviewing the dominant role of crack closure on modeling crack growth.

Analytical models for crack growth predictions based on plastic zone and deformation behavior ahead of the crack tip utilize, generally, crack driving force parameters such as ΔK and K_{max} to account for crack extension and load-interaction effects.[6,11] Recently, it has been shown[2] that crack driving force parameter ΔK^* [where $\Delta K^* = [\Delta K^+ K_{max}]^{0.5}$, ΔK^+ is the value of the positive part of the applied stress intensity factor (SIF) range, and K_{max} is the corresponding maximum value of the applied SIF] can account for effects of stress ratio on FCGR in aluminum alloys, resulting in a characteristic FCGR curve. This ΔK^* (essentially a geometric mean of ΔK^+ and K_{max}) approach does not make use of any crack opening load/stress data. Use of ΔK^* in predicting fatigue crack growth after a single overload in an aluminum alloy has met with reasonable success.[6]

In this investigation, an analytical method using crack driving force parameter ΔK^* is developed to predict FCGR under spectrum loading. The spectrum load sequence was approximated as an apparent CA sequence with σ_{max} and σ_{min} as rms maximum and minimum stresses of the load spectrum, respectively. The crack growth under this apparent CA load sequence was predicted using a crack growth law, defined in terms of crack driving force parameter ΔK^*. Fatigue

Received 27 September 2004; revision received 30 September 2005; accepted for publication 2 October 2005.

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crack growth experimental results of a vs N_b data were compared with crack growth predictions made by the proposed method.

Fatigue Crack Growth Prediction

The crack driving force parameter ΔK^* is defined as[2]

$$\Delta K^* = \left(\Delta K^+ K_{\max}\right)^{0.5} \tag{1}$$

where ΔK^+ is the value of the positive part of applied SIF range and $K_{\max}$ is the corresponding maximum value of the applied SIF. The FCGR $\mathrm{d}a/\mathrm{d}N$ is considered to be a function of the crack driving force parameter ΔK^*, and the complete sigmoidal shape of the FCGR curve is defined as[6]

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_1(\Delta K^*)^{C_2}\frac{\left[1-\left(\Delta K^*_{\mathrm{th}}/\Delta K^*\right)^2\right]}{\left[1-\left(\Delta K^*/C_3\right)^2\right]} \tag{2}$$

where C_1, C_2, C_3, and ΔK^*_{th} are constants determined from the FCGR experiments.

The fatigue crack growth prediction under a spectrum load sequence was performed by an rms approach.[12] The spectrum load sequence was considered to be an apparent constant amplitude load sequence with maximum and minimum stresses corresponding to the rms of maximum and minimum stresses of the spectrum sequence, respectively. The rms stresses were evaluated as[12]

$$\sigma^{\mathrm{rms}}_{\max} = \left[\frac{1}{N}\sum_{i=1}^{N_r}(\sigma_{\max})^2\right]^{0.5} \tag{3}$$

$$\sigma^{\mathrm{rms}}_{\min} = \left[\frac{1}{N}\sum_{i=1}^{N_r}(\sigma_{\min})^2\right]^{0.5} \tag{4}$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are maximum and minimum stresses in the spectrum sequence, N is the total number of $\sigma_{\max}$ or $\sigma_{\min}$ values, and N_r is the total number of reversals in the spectrum block. Then, for this apparent CA fatigue cycle sequence, the crack extension was calculated by a cycle-by-cycle approach using Eq. (2).

Experimental

The material used in this investigation was D16 (2024-T3 equivalent) aluminum alloy, which is mainly used in airframes. The standard chemical composition (in weight percent) of this material is as follows[6]: Cu, 3.8–4.9; Mg, 1.2–1.8; Mn, 0.3–0.9; Si, 0.5; Fe, 0.5; Zn, 0.3; and the balance in aluminum. The mechanical properties of the material determined from the tensile sheet specimens tested in longitudinal orientation were as follows: $\sigma_y = 347$ MPa, ultimate tensile strength $\sigma_{\mathrm{UTS}} = 460$ MPa, and percentage elongation = 12%.

Fatigue crack growth tests were performed using single-edge notched tension [SE(T)] specimens. Rectangular blanks with nominal dimensions of $W = 45$ mm, $L = 180$ mm, and $t = 1.5$ mm, having a central U-notch of length 2.5 mm and width 0.36 mm were fatigue precracked to a total crack length of approximately 3.80 mm. Crack growth tests were performed in a computer controlled, 100-kN, servohydraulic test machine under a fighter aircraft spectrum load sequence shown in Fig. 1. The total number of reversals in this load spectrum was 16,212. The maximum stress in the spectrum was 156 MPa, and the minimum stress was −39.6 MPa. Crack length was measured by a cellulose acetate replication method at the end of each block of loading to obtain crack length a vs the number of applied load blocks N_b data. The SIF K for an SE(T) specimen was expressed as

$$K = f(a/W)\sigma\sqrt{(\pi a)} \tag{5}$$

where

$$f(a/W) = 5/\{[20 - 13(a/W) - 7(a/W)^2]^{0.5}\} \tag{6}$$

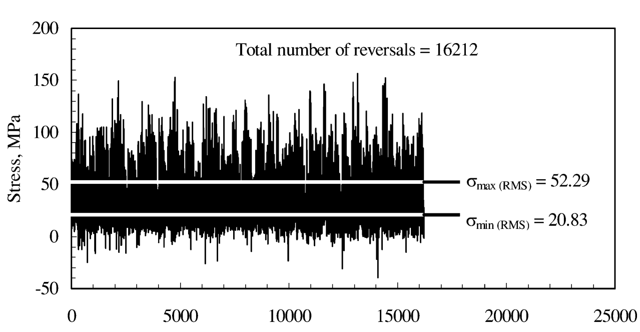

Fig. 1 Spectrum load sequence used in this investigation.

Results and Discussion

In an earlier investigation,[6] the CA fatigue crack growth tests at different stress ratios R in D16 aluminum alloy have been performed, and the FCGR as a function of crack driving force parameter ΔK^* has been obtained. The fatigue crack growth curve defined by Eq. (2) for D16 aluminum alloy[6] is shown in Fig. 2. The values of the constants in Eq. (2) for this material were taken from Ref. 6 and are as follows: $C_1 = 1.7589E{-}08$, $C_2 = 3.71$, $C_3 = 55$ MPa $\sqrt{}$ m, and $\Delta K^*_{\mathrm{th}} = 4.0$ MPa$\sqrt{}$ m. Thus, Eq. (2) with these values for various constants defines the characteristic FCGR curve for D16 aluminum alloy.

For the purpose of crack growth prediction, the spectrum sequence was approximated to be an apparent CA load sequence with $\sigma^{\mathrm{rms}}_{\max}$ and $\sigma^{\mathrm{rms}}_{\min}$ as explained earlier. The rms maximum and minimum stresses calculated using Eqs. (3) and (4) for the spectrum shown in Fig. 1 were as follows: $\sigma^{\mathrm{rms}}_{\max} = 52.29$ MPa and $\sigma^{\mathrm{rms}}_{\min} = 20.83$ MPa, which are shown as solid lines in Fig. 1.

The experimental results of crack length a vs the applied number of spectrum load blocks N_b obtained is shown in Fig. 3. Note that two tests were conducted to check the repeatability. As expected, crack

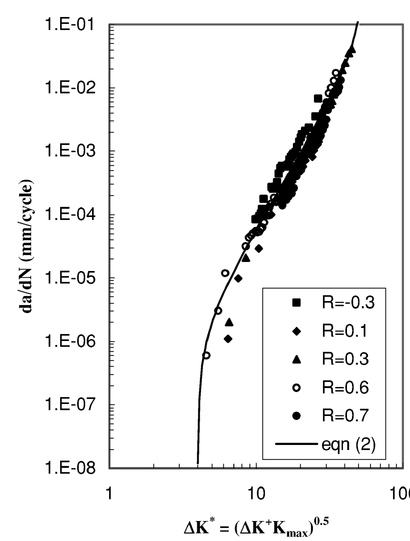

Fig. 2 FCGR data for D16 aluminum alloy.[6]

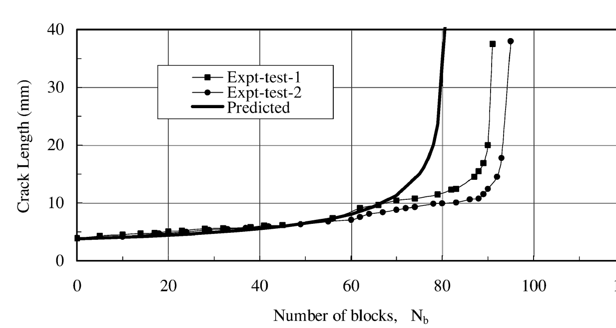

Fig. 3 Comparison of experimental and predicted fatigue crack growth in D16 aluminum alloy under spectrum load sequence.

length increased slowly in the initial stages and rapidly as the stress intensity factor increased at longer crack lengths. Almost similar fatigue crack growth behavior was observed in two tests performed. The average fatigue life of the specimen N_{expt} was found to be about 93 blocks.

Fatigue crack growth was predicted by the proposed method as explained earlier. Because one block of the spectrum load sequence consists of 16,212 reversals, application of 8106 cycles of apparent CA load cycles having $\sigma_{\max} = 52.29$ MPa and $\sigma_{\min} = 20.83$ MPa was considered as one block in the prediction procedure. Fatigue crack growth for each of the apparent CA cycles was calculated from Eq. (2). For the purpose of calculation, $W = 45$ mm, $t = 1.5$ mm, and the initial crack length $a_0 = 3.80$ mm were used. SIF was calculated from Eq. (5).

The crack growth behavior predicted by the foregoing method is shown in Fig. 3 along with experimental results. Note that there is quite a good correlation between experimental and predicted crack growth results at least up to approximately 75% of the total fatigue life. The fatigue life ratio $N_{\text{pred}}/N_{\text{expt}}$ was observed to be about 0.86. This is a conservative prediction and appears to provide a fairly good correlation with experimental results. Achieving life ratios between 0.5 and 2.0 is generally considered a good prediction of the total fatigue life.

The method adopted for fatigue crack growth life prediction in this investigation is considerably simple as compared to existing complex load-interaction models. Only a few input parameters shown in Eq. (2) need to be obtained for crack growth prediction. Simple computer programs incorporating this method can be easily developed, and the accuracy of crack growth prediction appears to be within the tolerance limit as well.

Note that the rms approach for crack growth prediction under spectrum loading should be used with caution. This approach is generally successful only when the loading sequence is purely random in nature[12] and then predominantly for tensile loads. For nonrandom loading sequence where relatively few high-load cycles cause long delays in crack growth, the rms approach may not be suitable.[12] Currently, conventional methods of crack growth prediction under the same spectrum load sequence via a crack closure concept is in progress. The results obtained from that approach will then be compared with rms approach described in this investigation. However, the results obtained in this investigation are quite encouraging because of the simple nature of the prediction procedure and the achievement of reasonable accuracy in crack growth life prediction.

Conclusions

Fatigue crack growth behavior in D16 aluminum alloy under a fighter aircraft spectrum load sequence was determined by both experimental and analytical procedures. The prediction was performed using crack driving force parameter ΔK^*. The rms approach was used to approximate the spectrum sequence into an apparent CA load sequence, and the crack growth was predicted by a cycle-by-cycle approach. A fairly good correlation was obtained between the experimental and predicted results. The fatigue life prediction was conservative by this method, and the predicted fatigue life ratio $N_{\text{pred}}/N_{\text{expt}}$ was about 0.86. The simple nature of the proposed prediction method and the reasonable accuracy in fatigue life prediction are quite encouraging. Further investigation comparing this method with a conventional method using a crack closure concept is underway and will be submitted for publication in the near future.

Acknowledgments

The author thanks A. R. Upadhya, Director, National Aerospace Laboratories (NAL), and P. K. Dash, Head, Structural Integrity Division (SID), NAL, for facilitating the conduct of this work. Useful discussions held with V. R. Ranganath and the assistance rendered by the MEL staff of the SID during the experimental part of this study are also acknowledged with thanks.

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